Reg. No.: .....

Name: .....

University of Kerala

U9168

Second Semester FYUGP Degree Examination, April 2025

Discipline Specific Core Course

MATHEMATICS

UK2DSCMAT101 - Integration and Multivariate Calculus

Academic Level: 100-199

Time: 1 Hour 30 Minutes(90 Mins.) **Max. Marks: 42**

Part A. 6 Marks.Time:6 Minutes.(Cognitive Level:Remember(RE)/Understand(UN)) Objective Type. 1 Mark Each.Answer all questions

Qn No.	Question	CL	CO
1	Write the formula for the volume of a solid bounded by two parallel planes perpendicular to the y-axis at $y = c$ and $y = d$.	RE	1
2	State Chain rule for Derivatives	RE	1
3	Let $f(x, y) = x^2y + 1$, find $f(1, 2)$.	UN	1
4	If C is the graph of a smooth vector valued function $r(t)$, then the unit normal vector to C at t is defined by $N(t) = \ldots$.	UN	1
5	Find the natural domain of the function $f(x, y) = \log xy$.	UN	1
6	Find $\bar{r}'(t)$ where $\bar{r}(t) = 6\hat{\imath} - \sin t\,\hat{\jmath}$.	UN	1

Part B.8 Marks.Time:24 Minutes.(Cognitive Level:Understand(UN)/Apply(AP))Short Answer. 2 marks each.Answer all questions

Qn No.	Question	CL	CO
7	Find the area of the region bounded above by $y = x + 6$, bounded below by $y = x^2$, and bounded on the sides by the lines $x = -2$ and $x = 3$.	UN	1
8	Evaluate the integral $\int_1^5 \frac{1}{\sqrt{2x-1}}\,dx$.	UN	1
9	Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if $z = e^{xy}\sin(5y^2)$.	AP	2
10	Find the volume of the solid that is obtained when the region under the curve $y = \sqrt{x}$ over the interval $[1, 4]$ is revolved about the x-axis.	AP	3

Part C. 28 Marks.Time:60 Minutes (Cognitive Level:Apply(AP)/Analyse(AN)/Evaluate(EV)/Create(CR)) Long Answer.7 marks each.Answer all 4 Questions choosing among options * within each question

Qn No.	Question	CL	CO
11	A) Find the slope of the sphere $x^2 + y^2 + z^2 = 1$ in the $y-$ direction at points $(\frac{2}{3}, \frac{1}{3}, \frac{2}{3})$ and $(\frac{2}{3}, \frac{1}{3}, \frac{-2}{3})$. OR B) Find the unit tangent vector $\mathbf{T}(t)$ and unit normal vector $\mathbf{N}(t)$ to the graph of $\mathbf{r}(t) = 4\cos t\ \mathbf{i} + 4\sin t\ \mathbf{j} + 3t\ \mathbf{k}$ at the point where $t = \dfrac{\pi}{2}$.	AP	2, 2
12	A) Suppose that $u = \sqrt{x^2 + y^2 + z^2}$, $x = \cos\theta$, $y = \sin\theta$, $z = \tan\theta$. Use the chain rule to find $\dfrac{du}{d\theta}$ when $\theta = \dfrac{\pi}{4}$ and then check the result by expressing u as a function of θ and differentiating directly. OR B) Find the volume of the solid that results when the region enclosed by $x = y^2$ and $x = y + 4$ is revolved about the y −axis.	AN	3, 2
13	A) Let $\bar{r}_1(t) = (tan^{-1}t)\hat{\imath} + (\sin t)\hat{\jmath} + t^2\hat{k}$ and $\bar{r}_2(t) = (t^2 - t)\hat{\imath} + (2t - 2)\hat{\jmath} + (\ln t)\hat{k}$. The graph of $\bar{r}_1(t)$ and $\bar{r}_2(t)$ intersect at the origin. Find the degree measure of the acute angle between the tangent lines to the graphs of $\bar{r}_1(t)$ and $\bar{r}_2(t)$ at the origin. OR B) a) Find area of the region that is enclosed between the curves $y = x^2$ and $y = x + 6$ b) Find the arc length of the curve $y = 3x^{\frac{3}{2}} - 1$ from $x = 0$ to $x = 1$	EV	3, 3
14	A) Locate all relative extrema and saddle points of $f(x, y) = x^2 + xy + y^2 - 6x$. OR B) Derive the formula for the volume of a sphere of radius r	CR	3, 2