Reg. No.: ....

Name: ....

Y8069

University of Kerala

Fourth Semester FYUGP Degree Examination, May 2026

Discipline Specific Core Course

MATHEMATICS

UK4DSCMAT200 - Introduction to Real Analysis and Multiple Integrals

Academic Level: 200-299

2024 Admission

Time: 1 Hour 30 Minutes(90 Mins.) **Max. Marks: 42**

Part A. 6 Marks.Time:6 Minutes.(Cognitive Level:Remember(RE)/Understand(UN)) Objective Type. 1 Mark Each.Answer all questions

Qn No.	Question	CL	CO
1	When we say a vector field $\mathbf{F}$ is conservative?.	RE	3
2	Define a sequence in $\mathbb{R}$.	RE	2
3	For $n \in \mathbb{N}$, let $I_n = [0, \frac{1}{n}]$. What is $\bigcup\limits_{n=1}^{\infty} I_n$?	UN	1
4	If C is a piecewise smooth curve from $(1,2,3)$ to $(4,5,6)$ then $\int\limits_C dx + 2dy + 3dz = \ldots\ldots$	UN	4
5	Sketch the region $R = \{(x,y) : 0 \leq y \leq x^2, 0 \leq x \leq 1\}$.	UN	3
6	Let $\mathbf{F}(x,y) = 2xy^3\mathbf{i} + (1 + 3x^2y^2)\mathbf{j}$. Determine whether $\mathbf{F}$ is a conservative vector field or not.	UN	4

Part B. 8 Marks.Time:24 Minutes.(Cognitive Level:Understand(UN)/Apply(AP))Short Answer. 2 marks each.Answer all questions

Qn No.	Question	CL	CO
7	If (x_n) converges to x and (y_n) converges to y, then prove that the sequence $(x_n + y_n)$ converges to $x + y$.	UN	2
8	If $t > 0$, prove that there exists $n_t \in \mathbb{N}$ such that $0 < \frac{1}{n_t}$.	UN	2
9	Let $\mathbf{F}(x,y) = e^y\mathbf{i} + xe^y\mathbf{j}$ denote the force field in the xy- plane. Verify the force field $\mathbf{F}$ is conservative on the entire xy- plane.	AP	4
10	Change the order of integration $\int_0^1 \int_x^1 dy\, dx$.	AP	3

Part C. 28 Marks.Time:60 Minutes (Cognitive Level:Apply(AP)/Analyse(AN)/Evaluate(EV)/Create(CR)) Long Answer.7 marks each.Answer all 4 Questions choosing among options * within each question

Qn No.	Question	CL	CO
11	A) Use Green's theorem to evaluate the integral $\int_C (3x^2 - 8y^2)dx + (4y - 6xy)\,dy$ where C is the boundary of the region defined by $y = \sqrt{x}$ and $y = x^2$. OR B) Show that $\int_C y\,\sin x\,dx - \cos x\,dy$ is independent of the path. Evaluate the integral along the line segment from $(0, 1)$ to $(\pi, -1)$.	AP	4, 4
12	A) Prove or disprove: If S is a subset of $\mathbb{R}$ that contains atleast two points and has the property 'if $x, y \in S$ and $x < y$, then $[x, y] \subseteq S$', then S is an interval. OR B) Prove or disprove: Sequence $S = (\sin n)$ is convergent.	AN	1, 1
13	A) Evaluate (a) $\int_0^{\ln 2} \int_0^1 xye^{y^2x}\,dydx.$ (b) $\int\int_R e^{-(x^2+y^2)}\,dA$, where R is the region enclosed by the circle $x^2 + y^2 = 4$. OR B) Evaluate the integrals (a) $\int_0^2 \int_0^x y\sqrt{x^2 - y^2}\,dydx$ (b) $\int_0^\pi \int_0^{\sin\theta} drd\theta$	EV	4, 4
14	A) (a) State the Divergence criteria. (b) Using (a) check the convergence of the sequence $\left(1,\ \frac{1}{2},\ 3,\ \frac{1}{4},\ 5,\ \frac{1}{6},\ \ldots\right)$.	CR	2, 2

Qn No.	Question	CL	CO
	(c) If X is a bounded sequence of real numbers that has the property that every convergent subsequence of X converges to $x \in R$. What can you say about the sequence X? Justify. OR B) Determine a sequence of real numbers such that every subsequence converges to the real number 5. Find the limit of the sequence. Is this sequence bounded? Why?		