**Reg. No.:** .............................

**Name:** ..................................

W7643

# University of Kerala

First Semester FYUGP Degree Examination, December 2025

Discipline Specific Core Course

**MATHEMATICS**

**UK1DSCMAT106 - Number Theory and Linear System of Equations**

Academic Level: 100-199

**2024-Admission**

**Time: 2 Hours(120 Mins)** **Max. Marks: 56**

**Part A.6 Marks:Time 5 Minutes.(Cognitive Level :Remember(RE)/Understand(UN)) Objective Type.1 mark each, Answer all questions**

| Qn No. | Question | CL | CO |
|---|---|---|---|
| 1 | Define eigenvalues of a square matrix. | RE | 4 |
| 2 | Write the formula for finding the sum of squares of first $n$ natural numbers. | RE | 3 |
| 3 | What is the maximum possible rank of a $4 \times 5$ matrix? | UN | 1 |
| 4 | Comment on it $50 = 1 \pmod 7$ | UN | 1 |
| 5 | State the condition for a linear system $AX = B$ of $m$ equations in $n$ unknowns to be consistent. | UN | 4 |
| 6 | Write two relatively prime integers. | UN | 3 |

**Part B.10 Marks.Time:20 Minutes (Cognitive Level:Understand(UN)/Apply(AP))Two-three sentences.2 marks each.Answer all questions**

| Qn No. | Question | CL | CO |
|---|---|---|---|
| 7 | Define symmetric and orthogonal matrices. | UN | 4 |
| 8 | Determine the rank of the matrix $A = \begin{bmatrix} 3 & 0 & 2 & 2 \\ -6 & 42 & 24 & 54 \\ 21 & -21 & 0 & -15 \end{bmatrix}$ | UN | 3 |
| 9 | If LCM of two numbers is 35 and their gcd is 375, then write the formula for finding their product and find the product. | AP | 2 |
| 10 | Using Cramer's Rule, solve the system of equations $4x - 6y = -11, -3x + 8y = 10$ | AP | 2 |
| 11 | Find gcd (25,35). | AP | 2 |

| Qn No. | Question | CL | CO |
|---|---|---|---|
| | | | |

**Part C.16 Marks.Time:35 Minutes.(Cognitive Level :Apply(AP)/Analyse(AN))Short Answer.4 marks each, Answer all 4 questions,choosing among options * within each question**

| Qn No. | Question | CL | CO |
|---|---|---|---|
| 12 | A)<br><br>Prove that $n(n+1)(2n+1)$ divisible by 6.<br><br>OR<br>B)<br><br>Solve the system of equations by Gauss elimination method<br>$x+2y+3z=1, 2x+3y+2z=2, 3x+3y+4z=1$. | AP | 2, 2 |
| 13 | A)<br><br>Find the eigenvalues and eigenvectors of the matrix $A=\begin{bmatrix} 8 & -4 \\ 2 & 2 \end{bmatrix}$<br><br>OR<br>B)<br><br>Examine whether the matrix $A=\begin{bmatrix} 1 & -3 & 3 \\ 0 & -5 & 6 \\ 0 & -3 & 4 \end{bmatrix}$ is diagonalizable. | AP | 1, 3 |
| 14 | A)<br><br>If $ac \equiv bc(\mathrm{mod}\, m)$ and $(c,m)=1$, prove that $a \equiv b(\mathrm{mod}\, m)$.<br><br>OR<br>B)<br><br>Find the canonical form of the quadratic $3x^2+2xy+3y^2$. Hence show that the equation $3x^2+2xy+3y^2-8=0$ represents an ellipse in $R^2$. | AN | 3, 3 |
| 15 | A)<br><br>Express $(28, 12)$ as a linear combination of 28 and 12.<br><br>OR<br>B)<br><br>Find the number of positive integers $\leq 3000$ and divisible by 3, 5, or 7. | AN | 3, 1 |

**Part D.24 Marks.Time: 60 Minutes.(Cognitive Level :Analyse(AN)/Evaluate(EV)/Create(CR)) Long Answer 6 Marks each.Answer all 4 questions choosing among options * within each question**

| Qn No. | Question | CL | CO |
|---|---|---|---|
| 16 | A)<br><br>Test the consistency of the system of equations<br>$x + y + z = 2, y + z = -2, 4y + 6z = -12, x + 6y + 8z = -12$. If the system is consistent, find its solution.<br><br>OR<br>B)<br><br>Find the highest power of 3 that divides 1001! | AN | 2, 2 |
| 17 | A)<br><br>Using the Euclidean algorithm, find the gcd of 4076 and 1024 and express (4076, 1024) as a linear combination of 4076 and 1024.<br><br>OR<br>B)<br><br>Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$. | EV | 2, 2 |
| 18 | A)<br><br>Prove that $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + n(n + 1) = \frac{1}{3}n(n + 1)(n + 2)$.<br><br>OR<br>B)<br><br>For what value of $\lambda$ and $\mu$ do the system of equations<br>$x + y + z = 6, x + 2y + 3z = 10, x + 2y + \lambda z = \mu$<br><br>has (i) no solution ; (ii) unique solution ; (iii) more than one solution. | EV | 3, 2 |
| 19 | A)<br><br>Find the matrix of transformation that diagonalize the matrix $\begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$. Also write the diagonal matrix.<br><br>OR<br>B)<br><br>Prove that no integer of the form $8n + 7$ can be expressed as a sum of three squares. | CR | 2, 1 |